

Sabbatical Leave Proposal for AY 2004-05
“Studies in Orthogonal Polynomials and Matrix Theory”

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I. Goals and Objectives:

- To deepen my understanding and expertise in my current research program, which is focused on the root and critical number distribution of certain families of real polynomials.
- To learn new mathematics in the areas of real analysis and linear algebra, particularly related to connections between orthogonal polynomials and matrix theory. Part of this goal includes an effort to solve remaining unsolved problems in my research by using new tools that may arise in this study. More detail regarding the motivation for this can be found in section III of this proposal. An additional aspect is seen in the next objective.
- To develop, formulate, and solve new research questions in the new mathematics I learn; some will be for my personal pursuit, while others will be for future joint work with undergraduates. Part of my proposed sabbatical will result in a research abstract that can be shared with future students: a document that offers appropriate background to the area of study, summarizes key results and concepts, poses new questions for consideration, and gives a detailed bibliography. More detail on this is given in section II.
- To gain additional perspective on my growth as a teacher by teaching a small number of courses at Hope College and comparing the work of colleagues there with my own and my colleagues here at Grand Valley. Hope College has a reputation for classroom excellence: I will seek to learn from colleagues there and reflect on how that (likely different) small college experience can improve and shape my future teaching at GVSU.
- To build stronger relationships between the mathematics departments at Hope and GVSU for future collaboration among the faculty at these two institutions. Possible areas for cooperative efforts here include shared colloquium speakers, more interaction centered on undergraduate research programs (both Hope and Grand Valley have NSF-funded REUs (Research Experiences for Undergraduates)), and enabling opportunities for joint scholarly or curricular work between faculty at the two schools.

II. Project Plan/Timetable

Pre-Sabbatical: In Winter 2004 I have been allocated 3 hours of alternate assignment for the purpose of continued development of joint research with undergraduates. I will spend some of this reserved time engaged in a preliminary reading of the journal articles given in the attached bibliography. Specifically, I will begin reading papers [1], [2], and [3] (see the bibliography, section VIII), as these show promise for being interesting and helpful for current problems I am working on (why these are potentially fruitful is explained in section III). Through this process, which will extend into summer 2004, I expect to be led to additional articles for reading as my sabbatical progresses in the fall.

Sabbatical:

In section III below, I discuss briefly how I conduct research through a combination of reading, problem-solving, problem-posing, and writing. A combination of these activities will occur (in various proportions) in each phase of my proposed sabbatical, as shown in the timetable below.

Early Fall 2004: I will continue working on existing open questions in my research, some of which are discussed in section III. In addition, the winter and summer reading of papers [1], [2], and [3] will provide a foundation for new learning of mathematics and the application of these tools to existing and new research problems. As study of new mathematics always leads to more reading (and more questions), I will read additional sources as my study directs. I expect that some of this will lead me to the books [9], [10], and [11]. I have read much of [10] and [11] in detail through my recent work, but more remains there to study, and [9] is a new source that merits my consideration.

As I work on existing questions and also develop new understanding through reading, I will also pose research questions for my own pursuit and make notes of other articles and topics that I discover that will provide additional opportunities for research with undergraduates.

I will also seek opportunities to give at least one talk on my research during the current academic year and will make plans to attend the Joint Mathematics Meetings in Atlanta, GA, in January 2005 in order to participate in the AMS sessions most closely related to my research.

Late Fall 2004: At this time, I will focus on one or two research questions that appear to be most tenable based on my most recent reading and learning. The majority of my research time will be spent being engaged in efforts to solve these. Along the way, I will read related sources for additional understanding, and continue to work on developing problems suitable for future undergraduate research.

Early Winter 2005: I will take some time in January to carefully assess my progress and chart a detailed plan for the remaining months ahead. Part of this will involve identifying additional reading to complete; it is likely that reading papers like [4]-[7] will be appropriate at this point. Based on my work in the fall, I will also make decisions about which research questions I will continue to pursue. In addition, I will begin by writing a summary of my reading and learning to date with the desire to make this report as accessible to undergraduates as possible.

Depending on my plans for the summer of 2005, I may also be engaged in writing an S3 grant for work with a GVSU student, or preparing a document for students in the GVSU REU (if that endeavor is funded and I am invited to participate).

The vast majority of my research time will be devoted to continued pursuit of solutions to research questions I will be addressing.

Late Winter 2005: As the academic year comes to a close, I will continue the work discussed above, but will begin to write more formally about my findings. In addition to the document in preparation for students who might join me in research, I will write a detailed technical report that summarizes my work. If the progress made is appropriate and significant, I will submit a version or portion of this document to a mathematics journal for review for publication.

Throughout the year: As I teach courses and interact with the mathematics faculty at Hope, I will endeavor to build relationships for future collaboration with them, not just for myself, but also for colleagues at GVSU. In addition, I will engage in teaching-related discussions with faculty there (possibly including some visits of others' classes) in order to reflect thoughtfully on my own and our department's (GVSU's) teaching endeavors.

Post Sabbatical:

Within the year or two following my sabbatical, I expect to engage in an undergraduate research project with one or more students on the subject of my study. I will also give a talk on my research in the mathematics department seminar and write the required report to fulfill Grand Valley's sabbatical requirements. Finally, I will attempt to publish at least one paper.

I will also share my teaching experiences at Hope with colleagues at GVSU in order that we as a department may have an external source of comparison for our work, and that we may learn from our colleagues at Hope. If appropriate, I will present a department seminar on this topic as well. Based on opportunities that arise in the course of the sabbatical, I will endeavor to promote at least one collaborative event between the math departments of Hope and GVSU.

III. Evidence of Preparation

During my six years at GVSU, I have been actively engaged in a research program devoted to the study of properties of the distributions of zeros and critical points of certain families of polynomials. This work has included supervision of two undergraduate research projects that have led to more than 10 joint or student presentations and 3 papers in review or in progress.

Throughout this time, I have regularly conducted four activities that have facilitated my work: **reading** of new journal articles and books, **solving** open problems, **posing** new problems for consideration, and **writing** about my results. These activities intertwine and inform one another in many ways; I therefore expect to do all of them (as noted in section II) during my proposed sabbatical.

During my first year at GVSU (1998-1999), I used much of my six hours of alternate assignment to read 6-10 papers that I had discovered late in my graduate career. Though these papers were related to my area of expertise, I had not had the opportunity to study them previously. This reading led me to gain new mathematical understanding and enabled me to develop research questions. These questions led me to yet more reading and study, and the combination of these efforts proved to be the foundation for two successful undergraduate research projects.

In Summer 2000, I continued my investigations of the mathematics related to these 6-10 papers on families of polynomials, and at that time invited GVSU undergraduate Matt Wells to do some reading with me. A year later we were granted a SURP (Summer Undergraduate Research Program) grant; our work in summer 2001 was very fruitful. We proved a large collection of results, with several of them regarding the distribution of critical numbers of the famous Chebyshev polynomials, specifically related to the so-called *ratio vector* of a polynomial. A sample of these results can be seen in the following theorem (which is part of a paper currently in review at the *Journal of Mathematical Analysis and Applications*):

Theorem. Let $T_n(x)$ denote the Chebyshev polynomial of degree n . Then:

1. $T_n(x)$ has a monotonically increasing ratio vector.
2. As the degree of $T_n(x)$ increases without bound, the ratios of the polynomial near the origin tend to the value $1/2$.
3. As the degree of $T_n(x)$ increases without bound, the j th ratio from the rightmost zero of $T_n(x)$ tends to the value $(4j+1)/8j$.

This result has led to a deeper understanding of properties of the Chebyshev polynomials. As Chebyshev polynomials belong to a broader family of orthogonal polynomials (called Jacobi polynomials), it is natural to seek to extend results like this to other polynomials in this larger collection, such as Legendre polynomials, or Chebyshev polynomials of the second, third, and fourth kinds.

In Summer 2003, I continued to work on these extensions as a member of the GVSU NSF REU; there I was faculty mentor to two students, Jennifer Miller of Bucknell University and Benjamin Vugteveen of GVSU. Our work led to some partial results (as well as to some significant theorems on a problem of a different nature), but also showed that additional tools are necessary. In the course of our pursuits, I consistently ran into papers like [1]-[3] that referred to matrix methods for better understanding or alternative approaches to the zero-distribution of orthogonal polynomials. As we are interested in the distribution of both critical numbers and zeros, this appears to be a promising line of work, and partially motivates this proposal. In addition, the connections between matrix theory and polynomial zero bounds seem to provide additional potential for generating interesting questions for undergraduate research projects.

Finally, I should discuss my preparation for teaching some classes while simultaneously doing research, as well as for growing as a teacher and building bridges during my time at Hope. One of the things I enjoy most about Grand Valley is its balanced environment where both teaching and scholarship are valued; I believe that I am better at both these endeavors when I conduct them alongside one another. While I could have a satisfactory experience doing research full time for one semester, I would much prefer to do so half time (or slightly more) for an entire academic year. This more balanced arrangement will, I think, enable me to accomplish more in the long run. To support this claim, I note that in summer 2001 (during the SURP grant) I taught two three credit classes over twelve weeks (one met 3 days a week for the first six weeks, the other 2 days a week for the second six). This schedule helped me to have variety (with balance) in my days and be even more focused in my research.

In addition, I have learned a great deal about how to be a better teacher from my colleagues at Grand Valley. Regular interactions with talented instructors here have helped me reflect and grow and gain greater effectiveness in my own classroom. I would greatly enjoy having the opportunity to learn similarly from the mathematics faculty at Hope. Moreover, during my time at GVSU I have consistently worked to build relationships and connections among institutions through my involvement with Michigan Project NExT and the Michigan Undergraduate Mathematics Conference, as well as in my service as the GVSU mathematics colloquium organizer where I often invite colleagues from other schools to speak to our faculty. I would use the opportunity of working at Hope to build closer relationships with faculty at a nearby college to open doors for future joint initiatives in various settings.

IV. Arrangements with people or other institutions

Janet Andersen, chair of the mathematics department at Hope College, has made a verbal offer of a half-time position for me for the 2004-05 academic year. This position will cover the remaining half of my salary, provide me with office space, and may offer some additional travel support. I will be able to use my own GVSU laptop computer for most of my technology needs, though they will provide a PC for me. The position will require me to teach one or two classes per term (at most three for the year, with only one prep per term) and will not carry any service responsibilities. Where appropriate, I will take advantage of opportunities to participate in department colloquia, as well as related curricular discussions to any courses I teach.

V. Letters of support

from Janet Andersen and Paul Fishback (are attached)

VI. Benefits to other units

This project will enable me to continue to contribute to the overall climate of undergraduate research projects at GVSU, thereby strengthening the community of scholars we seek to build not just in our department, but across the sciences (and indeed all disciplines) at our university. One direct consequence

may be enhancing the contributions I will make to our department's REU, whose impact reaches mathematics students at universities other than GVSU.

The development of strong ties between our department and the one at Hope College may lead to similar cooperative endeavors between other units at the two schools. Such cooperation is good for not just the mathematics community, but for the community of higher education in general.

As our department teaches students from across the university, the opportunities for me to grow personally as a teacher in my sabbatical, along with the opportunity to compare and critique our department's work with that of another college, will potentially improve our instruction here at Grand Valley. As such, there is promise of future benefit for students in my own classroom, as well as those taking mathematics courses in general, as we may implement effective teaching strategies being used by colleagues at Hope.

VII. Curriculum Vita

Is attached.

VIII. Bibliography

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