Mathematics Faculty Sabbatical Report

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Semester(s) During Which Sabbatical Was Taken: Winter 2004

(Descriptive) Title of Project: *Developing Interactive Maple Packages for Modern Algebra (MTH 310)*

During my sabbatical I began developing *Maple* software and activities for use in Modern Algebra (Math 310). My software was based on a software package developed by Al Hibbard and Ken Levasseur. Hibbard and Levasseur's software is written in *Mathematica*, which they use in their Modern Algebra classes. Since a direct translation of *Mathematica* code into *Maple* code was not practical due to differences in programming paradigms, my software focuses on translating the ideas rather than the actual code into *Maple* software.

At the end of the summer of 2004 I had a working *Maple* **module** constructor that creates what Hibbard and Levasseur call a *Ringoid*, which is a set with two binary operations ("addition" and "multiplication", which generally do not correspond to the usual addition and multiplication, and in fact may or may not be well defined). Using my software, as student can experiment to determine if the ringoid is in fact a ring by checking the properties of the ringoid (for example, to determine whether "addition" is a commutative operation in the ringoid). I had also begun developing (and have since completed) graphical interfaces that allow students to create a finite set of elements and then use interactive Cayley tables to define and refine binary operations on the set of elements. The package creates a ringoid as a *Maple* **module**, which means that in addition to using the graphical Cayley table, the student can use the familiar "+" and "." notation for addition and multiplication inside *Maple*'s **use** environment to carry out calculations in the ringoid.

I have used, and will continue to use, the *Ringoids* package in Math 310. For example, I developed a computer activity in which students started with a set of four elements and attempted to use the interactive Cayley tables to create multiplication and addition tables for the set so that all of the ring axioms were obeyed. (This was accomplished by modifying the Cayley tables, then exiting the *maplet* to check the ring axioms. In the future, I hope to be able to have the students be able to check these properties by pressing buttons in the *maplet*.) Along the way, students develop and check hypotheses regarding which ring axioms are responsible for which derivative properties. For example, students may discover that distributivity of multiplication over addition is necessary for the zero-product property; this gives students insight into a proof of the zero product property.

I intend to continue using and extending the ringoids package. I have already begun work on developing a **Morphoid** module, which will eventually allow students to define functions from one ring to another, modify the function, and test whether the function is a homomorphism or even an isomorphism of rings.

A current version of the Ringoids package is available for use by any faculty member or student in the computer labs where it can be loaded using the standard *Maple* "with" command, "with(Ringoids):".