The 58th Midwestern Graph Theory Conference
Schedule and Abstracts*

October 6–7, 2017

Schedule

All talks take place in Kindschi Hall.

Friday, October 6

3:00–4:00	*Registration, Atrium*
4:00–4:50	Plenary Talk: David Galvin, Room 1101
5:00–7:00	Pizza Social and Undergraduate Poster Session, rooms 2207 & 2213

Saturday, October 7

8:00–8:45	*Registration, Atrium*			
8:45–9:00	Welcome, Room 1101			
9:00–9:50	Plenary Talk: Douglas B. West, Room 1101			
9:50–10:10	*break*			
	Room 2207	Room 2213	Room 2219	Room 2225
10:10–10:30	Blumenthal	Rorabaugh	Reinhart	Kikas
10:30–10:50	McKee	Mudrock	Bjorkman	Boats
10:50–11:10	*break*			
11:10–11:30	English	Machacek	Phillips	Krueger
11:30–11:50	Li	Asplund	Curl	McCourt
11:50–12:10	Fallon	Engbers	Carlson	DeBiasio
12:10–1:30	*lunch*			
1:30–2:20	Plenary Talk: David Galvin, Room 1101			
2:30–2:40	*photo*			
2:40–3:00	Johnston	Heath	McKenney	Olejniczak
3:00–3:20	Neidinger	Liu	Groothuis	Parker
3:20–3:40	Schulte	Loeb	Bucher	
3:40–4:00	*break*			
4:00–4:20	Fernandez	Perry	Weakley	
4:20–4:40	Kirsch	Byers	Păcurar	
4:40–5:00	Vandell	Hart	Rombach	

*Organizing Committee: Lauren Keough, Benjamin Reiniger, Michael Santana, Taylor Short

Plenary Talks

Equilateral and almost-equilateral sets in $\mathbb{R}^n$

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An *equilateral set* is a set of points any two of which are the same distance apart (so the vertices of an equilateral triangle form an equilateral set in the plane, and the vertices of the regular tetrahedron form one in space). It's well known that the largest equilateral set in n-dimensional space has size $n+1$. While this is a geometric fact, it admits a lovely linear algebra proof.

In 1962 Danzer and Grünbaum asked how large a set can be if it is *almost* equilateral — pairs of points are close to the same distance apart. They made the reasonable conjecture that the largest almost-equilateral set is never much larger than the largest equilateral set. Twenty years later, Erdős and Füredi spectacularly disproved this using a probability argument.

Erdős and Füredi's work was highly non-constructive. Nicely illustrating that one can never predict where a mathematical problem is going to go next, recently Zakharov, a high-school student in Moscow, revisited the linear algebra approach, and improved on Erdős and Füredi's result — this time in a completely constructive way.

I'll talk about some of this work, and mention a few nice open questions.

Reconstruction from the deck of k-vertex induced subgraphs

Douglas B. West
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The k-deck of a graph G is its multiset of subgraphs induced by k vertices; we ask when the k-deck determines G. Let $n = |V(G)|$. The famous Reconstruction Conjecture is that the $(n-1)$-deck determines G when $n \geq 3$. Always the k-deck determines the $(k-1)$-deck, so the natural question is to find the least k such that the k-deck determines G.

An easy first result is that a complete r-partite graph is determined by its $(r+1)$-deck. We then generalize a result of Bollobas by showing that for $l = (1-o(1))n/2$, almost every graph G is determined by various sets of $\binom{l+2}{2}$ subgraphs with $n-l$ vertices. However, when $l = n/2$, the entire $(n-l)$-deck does not always determine whether G is connected (it fails for n-vertex paths). We strengthen a result of Manvel by proving for each l that when n is sufficiently large (at least l^{l^2}), the $(n-l)$-deck determines whether G is connected ($n \geq 25$ suffices when $l = 3$). Finally, for every graph G with maximum degree 2, we determine the least k such that G is reconstructible from its k-deck, which involves extending a result of Stanley.

These results are joint work with Hannah Spinoza.

Plates, olives, and Morse theory

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Morse theory seeks to understand the shape of a manifold by studying smooth functions on it. Although a smooth function on a smooth manifold is an inherently continuous object, the study of Morse functions can lead to interesting and non-trivial combinatorial questions.

For example, in 2006 Arnold asked how many Morse functions the sphere admits. Fixing the dimension of the sphere, the number of critical points of the function and some natural notion of equivalence of functions, this becomes a discrete question, and has connections to well-known combinatorial objects such as alternating permutations, Catalan paths and Young's lattice.

One version of the problem leads to Nicolaescu's *game of plates and olives*. Start with an empty table, and at each step either add an empty plate, or add an olive to a plate, or eat an olive from a plate, or remove an empty plate, or combine the olives from two plates and remove one of the plates. The number of games of length $2n$ that start and end with an empty table is (essentially) the number of Morse functions on S^2 with n saddle points, up to a notion of topological equivalence.

We have identified the growth rate of this quantity, answering a question of Nicolaescu. It's an area where plenty more work remains to be done, though.

Joint work with Teena Carroll, Emory & Henry College.

Contributed Talks

The Minimum Coprime Number and Graph Operations

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Joint work with: Brad Fox (Austin Peay State University).

Graph labeling is a large topic of research as evidenced by the seminal 430+ page survey on the topic by Joe Gallian. In this talk, I will focus on one small sector of this larger topic: prime labelings. We say a graph has a *prime labeling* if we can label the vertices of a graph of order n with distinct labels from $\{1, 2, \ldots, n\}$ so that the labels on adjacent vertices are relatively prime.

Many graphs are not prime, including all but one of the complete graphs. To be as inclusive as possible, we will primarily discuss coprime labelings in this talk. A *coprime labeling* of a graph is the same thing as a prime labeling except we use the labels $\{1, 2, \ldots, m\}$ for some $m > n$ instead of the labels $\{1, 2, \ldots, n\}$. To make this more interesting, we care about making m as small as possible and call a labeling of the vertices of a graph using distinct m positive integers (m is minimized) with relatively prime adjacent vertex labels a *minimum coprime labeling*. Finding this minimum coprime labeling will be our main focus in this talk.

Ordered multiplicity inverse eigenvalue problem for graphs on six vertices

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Joint work with: John Ahn, Christine Alar, Steve Butler, Joshua Carlson, Audrey Goodnight, Haley Knox, Casandra Monroe, and Michael C. Wigal.

For a graph G, we associate a family of real symmetric matrices, $\mathcal{S}(G)$, where for any $M \in \mathcal{S}(G)$, the location of the nonzero off-diagonal entries of M are governed by the adjacency structure of G. The ordered multiplicity *Inverse Eigenvalue Problem of a Graph (IEPG)* is concerned with finding all attainable ordered lists of eigenvalue multiplicities for matrices in $\mathcal{S}(G)$.

For connected graphs of order six, we offer significant progress on the IEPG, as well as a complete solution to the ordered multiplicity IEPG. We also show that while $K_{m,n}$ with $\min(m, n) \geq 3$ attains a particular ordered multiplicity list, it cannot do so with arbitrary spectrum.

On secure-domination in graphs

Adam Blumenthal
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Joint work with: Peter Dankelmann and Fadekemi Janet Osaye (University of Johannesburg).

Let G be a graph and S a non-empty set of vertices of G. Let $N(S)$ be the set of vertices adjacent to some vertex in S. An attack on S is a mapping which maps each vertex $v \in N(S) - S$ to a neighbour of v in S. A defence of S is a mapping that maps each vertex $w \in S$ to a neighbour of w in S or to w. We say that a defence of S thwarts and attack on S if every vertex in S has at least as many defenders as attackers. A set S is secure if for every attack on S there exists a defence of S that thwarts the attack.

A secure-dominating set of G is a set S that is secure as well as dominating, i.e., every vertex of G is in $S \cup N(S)$. The smallest cardinality of a secure-dominating set of G is the secure-domination number of G, denoted by $\gamma_s(G)$.

It is not known if there exists a constant c with $c < 1$ such that

$$\gamma_s(G) \leq cn$$

for all connected graphs G of order n. In this paper talk we determine the maximum secure-domination number of trees with given order, and obtain that if we restrict ourselves to tree, then the above bound on γ_s holds with $c = \frac{2}{3}$ and this is best possible.

We also show, by constructing suitable examples, that if such a constant c exists for r-connected graphs, then it is at least $\frac{\lceil \frac{r}{2} \rceil + 1}{2\lceil \frac{r}{2} \rceil + 1}$.

Finally, we provide an upper bound for the secure domination number in arbitrary graphs.

Pansophical Classes of Graphs

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Joint work with: Lazaros Kikas.

(This talk is a followup to the previous talk by L. Kikas.)

Now that the concept of pansophy is defined, we explore the question: "which classes of graphs are **pansophical**?" That is, for an entire class of graphs, can the pansophies of each be quickly determined, either by an explicit formula, or by a polynomial-time algorithm? We show that the classes P_n and C_n are pansophical via an explicit formula in n, while $K_{m,n}$ is pansophical via algorithm. We also comment on which other graph classes show promise.

Quiver Mutation: Using directed graphs to understand topological surfaces

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In this short talk, we will explore a process of mutating a directed graph to produce a new directed graph. This procedure has applications to a variety of mathematical areas including mathematical physics, topology, number theory, geometry, and representation theory. In this talk we will briefly discuss how this process can be used to understand the topology of marked surfaces.

Rainbow Colorings In Hamiltonian-Connected Graphs

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Joint work with: Zhenming Bi and Ping Zhang.

A rainbow coloring of a connected graph G is an edge coloring of G, where adjacent edges may be colored the same, with the property that for every two vertices u and v of G, there exists a $u - v$ rainbow path (no two edges of the path are colored the same). The minimum number of colors in a rainbow coloring of G is the rainbow connection number of G. This topic has been studied by many. We study this concept in Hamiltonian-connected graphs from a different point of view. Recent results and problems in this area of research are presented.

Throttling for Positive Semidefinite Zero Forcing

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Joint work with: J. Kritschgau, L. Hogben, K. Lorenzen, M. Ross, S. Selken, and V. Martinez.

The *color change rule* for zero forcing in a graph G is that a blue vertex v can force a white vertex w to become blue if and only if w is the only white neighbor of v in G. If B_0 is the initial set of blue vertices, let B_{i+1} be the set of blue vertices in G after the color change rule is applied to every vertex in the set B_i. Such a set B_0 is a *zero forcing set* in G if there exists a n such that $B_n = V(G)$. The *zero forcing number of* G is the size of a minimum zero forcing set. The *propogation time* for a zero forcing set B_0, $pt(G, B_0)$, is the smallest n such that $B_n = V(G)$. The *throttling number* of G for zero forcing is the minimum of $|B_0| + pt(G, B_0)$ where B_0 ranges over all zero forcing sets of G. Throttling for zero forcing has been studied by Butler and Young, *Australasian Journal of Combinatorics*, 2013. Positive semidefinite (PSD) zero forcing is a variant in which the color change rule is applied to each $G[B_0 \cup C_i]$ where $C_1, C_2, \ldots, C_k$ are the components of $G - B_0$. This talk will present results on throttling for PSD zero forcing.

Generalized Petersen graphs with maximum nullity equal to zero forcing number

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Joint work with: Joe Alameda, Armando Grez, Leslie Hogben, O'Neill Kingston, Alex Schulte, Derek Young, and Michael Young.

The maximum nullity of a simple graph G, denoted $\mathrm{M}(G)$, is defined to be the largest possible nullity over all symmetric real matrices whose ijth entry is nonzero exactly when $\{i, j\}$ is an edge in G for $i \neq j$, and the iith entry is any real number. The zero forcing number of a simple graph G, denoted $\mathrm{Z}(G)$, is the minimum number of blue vertices needed to force all vertices of the graph blue by applying the color change rule. The motivation for this research is the longstanding question of characterizing graphs G for which $\mathrm{M}(G) = \mathrm{Z}(G)$. The following conjecture was proposed at the 2017 AIM workshop *Zero-forcing and its applications:* If G is a bipartite 3-semiregular graph, then $\mathrm{M}(G) = \mathrm{Z}(G)$. A counterexample was found by J. C.-H. Lin but questions remained as to which bipartite 3-semiregular graphs have $\mathrm{M}(G) = \mathrm{Z}(G)$. This talk concentrates on one family of graphs known as the Generalized Petersen graphs. These graphs are 3-regular and are only bipartite in specific cases. We were able to establish $\mathrm{M}(G) = \mathrm{Z}(G)$ for certain Generalized Petersen graphs. Determining if the equivalence of the maximum nullity and zero forcing number holds for the entire family remains to be shown.

Spanning trees with few branch vertices

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Joint work with: Allan Lo (University of Birmingham, UK).

A branch vertex in a tree is a vertex of degree at least three. We prove that, for all $k \geq 1$, every connected graph on n vertices with minimum degree at least $(\frac{1}{k+3} + o(1))n$ contains a spanning tree having at most k branch vertices. Asymptotically, this is best possible and solves a problem of Flandrin, Kaiser, Kužel, Li and Ryjáček, which was originally motivated by an optimization problem in the design of optical networks.

Extremal colorings and independent sets in k-chromatic graphs

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Joint work with: Aysel Erey.

Given a family of graphs, which graph in the family has the most number of proper colorings (vertex colorings where adjacent vertices receive different colors)? Tomescu answered this question for n-vertex k-chromatic graphs, and conjectured an answer for n-vertex k-chromatic connected graphs. Recently, Knox and Mojar announced a forthcoming proof of Tomescu's conjecture.

A color class in a proper coloring forms an *independent set* of vertices, or set of pairwise non-adjacent vertices. Which graph in a family of graphs has the most number of independent sets? We present some results in the family of n-vertex k-chromatic graphs with several different connectivity requirements. Numerous open questions remain.

Saturation for Berge Hypergraphs

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Joint work with: Jessica Fuller, Nathan Graber, Pamela Kirkpatrick, Abhishek Methuku, and Eric C. Sullivan.

Let H be a k-uniform hypergraph, and F be a simple graph on the same vertex set. We say H is *Berge-F* if there exists a bijection $f : E(F) \to E(H)$ such that for each $e \in E(F)$, we have $e \subset f(e)$. If there exists a subhypergraph of H that is Berge-F we say that H contains Berge-F. A hypergraph, H is *Berge-F-saturated* if H does not contain Berge-F but $H+e$ contains Berge-F for every edge $e \in E(\overline{H})$. The k-uniform *saturation number* of Berge-F, denoted $sat_k(n, \text{Berge-}F))$, is the minimum number of edges in a k-uniform hypergraph H such that H is Berge-F-saturated. In this talk we will explore the saturation numbers of many Berge hypergraphs.

Criticality of Counterexamples to Edge-hamiltonicity on the Klein Bottle

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Tutte and Thomas and Yu proved that 4-connected planar and projective-planar graphs, respectively, are Hamiltonian. Grünbaum and Nash-Williams conjecture that 4-connected toroidal and Klein bottle graphs are hamiltonian. Thomassen constructed counterexamples to edge-hamiltonicity of four-connected toroidal and Klein bottle graphs. Ellingham and Marshall contribute to the characterization of four-connected toroidal graphs in which some edge is not on a hamilton cycle, showing a sort of criticality of Thomassen's counterexamples and their generalizations. We contribute to the characterization of 4-connected Klein bottle graphs that have some edge not on a hamilton cycle, showing a criticality similar to that in Ellingham and Marshall's toroidal graphs.

Maximal Planar Subgraphs of Fixed Girth in Random Graphs

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Joint work with: Nicholas Sieger and Michael Tait.

In 1991, Frieze and Bollobás showed that the threshold for $G_{n,p}$ to contain a spanning maximal planar subgraph is very close to $p = n^{-1/3}$. In this paper, we compute similar threshold ranges for $G_{n,p}$ to contain a maximal bipartite planar subgraph and for $G_{n,p}$ to contain a maximal planar subgraph of fixed girth g.

Maximizing Cliques in Shellable Clique Complexes

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Maximizing cliques in graphs under certain restrictions on graph parameters has been a fruitful area of research since Tuŕan. Since this is equivalent to maximizing the number of faces in the clique complex of the graph, it is natural to extend the available parameters to include properties of simplicial complexes. Shellability is a notable property of simplicial complexes which means that the complex can be assembled or disassembled in a particularly nice manner. In this talk, we will find the maximum number of faces in the clique complex of a graph subject to the conditions that the graph has a given maximum degree and its clique complex is shellable. We will also exhibit a graph achieving this upper bound.

Majestic t-Tone Colorings

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Joint work with: Ping Zhang.

For integers t and k with $1 \leq t < k$, let $[k]_t$ denote the set of t-element subsets of $[k] = \{1, 2, \ldots, k\}$. For a connected graph G, let $c : E(G) \to [k]_t$ be an edge coloring of G where adjacent edges may be colored the same. Then c induces a vertex coloring c' of G obtained by assigning to each vertex v of G the union of the sets of colors of the edges incident with v. The edge coloring c is a majestic t-tone k-edge coloring of G if the induced vertex coloring c' is a proper vertex coloring of G. The minimum positive integer k for which a graph G has a majestic t-tone k-edge coloring is the majestic t-tone index of G. We present recent results and open questions in this area of research.

A (5,5)-coloring of K_n with few colors

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Joint work with: Alex Cameron (University of Illinois at Chicago).

A (p, q)-coloring of a graph G is an edge-coloring of G in which each p-clique contains edges of at least q distinct colors. We denote the minimum number of colors needed for a (p, q)-coloring of the complete graph K_n by $f(n, p, q)$. In this talk, we will describe an explicit $(5, 5)$-coloring of K_n which proves that $f(n, 5, 5) \leq n^{1/3+o(1)}$ as $n \to \infty$, improving the best known probabilistic upper bound of $O(n^{1/2})$ given by Erdős and Gyárfás.

Rainbow Turán Numbers for Paths and Forests of Stars

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Joint work with: Cory Palmer and Amites Sarkar.

For a fixed graph F, we consider the maximum number of edges in a properly edge-colored graph on n vertices which does not contain a rainbow copy of F, that is, a copy of F all of whose edges receive a different color. This maximum, denoted by $ex^*(n; F)$, is the rainbow Turán number of F, and its systematic study was initiated by Keevash, Mubayi, Sudakov and Verstraëte [*Combinatorics, Probability and Computing* **16** (2007)]. In this talk, we look at $ex^*(n; F)$ when F is a forest of stars, and consider bounds on $ex^*(n; F)$ when F is a path with l edges, disproving a conjecture in the aforementioned paper for $l = 4$.

Graph Pansophy

Lazaros Kikas
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Joint work with: Jeffe Boats.

Given a graph G, we are interested in finding disjoint paths for a given set of distinct pairs of vertices. In this talk we formally define a new parameter, the **pansophy of** G**,** $\Phi(G)$ in the context of the disjoint path problem. We will discuss how this parameter may be computed and its usefullness in the studying the optimality of routing algorithms. We then discuss the pansophy of instances of several different classes of graphs. We close with future research directions.

Many triangles with few edges

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Joint work with: A.J. Radcliffe.

Extremal problems concerning the number of independent sets or complete subgraphs in a graph have been well studied in recent years. Cutler and Radcliffe proved that among graphs with n vertices and maximum degree at most r, where $n = a(r + 1) + b$ and $0 \leq b \leq r$, $aK_{r+1} \cup K_b$ has the maximum number of complete subgraphs, answering a question of Galvin. Gan, Loh, and Sudakov conjectured that $aK_{r+1} \cup K_b$ also maximizes the number of complete subgraphs K_t for each fixed size $t \geq 3$, and proved this for $a = 1$. Cutler and Radcliffe proved this conjecture for $r \leq 6$.
We investigate a variant of this problem where we fix the number of edges instead of the number of vertices. We prove that $aK_{r+1} \cup \mathcal{C}(b)$ maximizes the number of triangles among graphs with m edges and any fixed maximum degree $r \leq 8$, where $\mathcal{C}(b)$ is the colex graph on b edges, $m = a\binom{r+1}{2} + b$, and $0 \leq b < \binom{r+1}{2}$.

Hamiltonian Cycles in k-Partite Graphs

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Joint work with: Louis DeBiasio, Dan Pritikin, and Eli Thompson.

Chen, Faudree, Gould, Jacobson, and Lesniak determined a minimum degree threshold for which a balanced k-partite graph has a Hamiltonian cycle, extending a result of Moon and Moser about Hamiltonian cycles in balanced bipartite graphs. However, when $k \geq 3$ a k-partite graph is not necessarily balanced. We determine some minimum degree thresholds that generalize the Moon and Moser result to not-necessarily-balanced k-partite graphs and determine when these conditions are asymptotically tight. We perform a stability analysis by showing that a graph obeying the degree conditions is either a robust expander, or else has a Hamiltonian cycle directly.

On the number of linear hypergraphs of large girth

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Joint work with: József Balogh.

An r-uniform *linear cycle* of length ℓ, denoted by C_ℓ^r, is an r-graph with edges $e_1, \ldots, e_\ell$ such that for every $i \in [\ell-1]$, $|e_i \cap e_{i+1}| = 1$, $|e_\ell \cap e_1| = 1$ and $e_i \cap e_j = \emptyset$ for all other pairs $\{i,j\},\ i \neq j$. For a linear r-graph H, the *linear Turán number* of H, denoted by $\mathrm{ex}_L(n,H)$, is the maximum number of edges among linear r-graphs on n vertices which contain no H as a subgraph. Collier-Cartaino, Graber and Jiang proved that $\mathrm{ex}_L(n, C_\ell^r) = O\left(n^{1+\frac{1}{\lfloor \ell/2 \rfloor}}\right)$ for all $r \geq 3$ and $\ell \geq 4$. Inspired by this development on linear Turán number, we prove that the number of linear hypergraphs without C_4^r is $2^{O(n^{3/2})}$. Further, for every $r \geq 3$ and $\ell \geq 4$, we show that the number of linear r-graphs of girth at least ℓ is $2^{O(n^{1+1/\lfloor \ell/2 \rfloor})}$. Our method comes from Kleitman and Winston, and Kohayakawa, Kreuter and Steger.

Packing chromatic number of cubic graphs

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Joint work with: József Balogh and Alexandr Kostochka.

A *packing k-coloring* of a graph G is a partition of $V(G)$ into sets $V_1, \ldots, V_k$ such that for each $1 \leq i \leq k$ the distance between any two distinct $x, y \in V_i$ is at least $i+1$. The *packing chromatic number*, $\chi_p(G)$, of a graph G is the minimum k such that G has a packing k-coloring. Sloper showed that there are 4-regular graphs with arbitrarily large packing chromatic number. The question whether the packing chromatic number of subcubic graphs is bounded appears in several papers. We answer this question in the negative. Moreover, we show that for every fixed k and $g \geq 2k+2$, almost every n-vertex cubic graph of girth at least g has the packing chromatic number greater than k.

Entire Colorability for a Class of Plane Graphs

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Joint work with: Axel Brandt, Michael Ferrara, Nathan Graber, and Stephen Hartke.

A plane graph G is *entirely k-colorable* if every element in the set of vertices, edges, and faces of G can be colored from $1, \ldots, k$ so that every two adjacent or incident elements have distinct colors. In 2011, Wang and Zhu asked if every simple plane graph G, other than K_4, is entirely $(\Delta(G)+3)$-colorable. In 2012, Wang, Mao, and Miao answered in the affirmative for simple plane graphs with $\Delta(G) \geq 8$. We show that every loopless plane multigraph with $\Delta(G) = 7$, no 2-faces, and no two 3-faces sharing an edge is entirely 10-colorable.

Plurigraph coloring and scheduling problems

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We define an object called a plurigraph which parameterizes a certain class of the scheduling problems defined by Breuer and Klivans. Proper coloring in plurigraphs encompasses proper coloring in graphs and hypergraphs. Oriented coloring and acyclic coloring are also special cases of plurigraph coloring. We will give a deletion-contraction formula which is valid for the chromatic polynomial and chromatic symmetric function in noncommuting variables.

The Dishonest Salesperson Problem

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A salesperson's office is located on a vertex v of a connected, unweighted graph G with n vertices, $n-1$ of which are customers. The salesperson leaves the office, visits each customer exactly once and returns to the office. Because a profit is made on mileage allowance, the salesperson wants to maximize the distance traveled. What is that maximum distance, and how many different such trips are there? I will present the results for the hypercube.

The Politics of Universal Double-Crossing

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After apologizing for the title, I will discuss how a 1977 characterization of distance-hereditary graphs, expressed in terms of crossing chords of cycles, leads to a strengthening (and then an even further strengthening) of distance-hereditary graphs, now in terms of double-crossed chords of cycles. Both of the stronger concepts have multiple characterizations with interesting parallel comparisons.

Asymptotic density of monochromatic subgraphs

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Joint work with: Louis DeBiasio.

Ramsey's theorem implies that every countably infinite graph can be found as a monochromatic subgraph in any 2-coloring of the complete graph on the natural numbers. A natural question to ask is: how dense (in the natural numbers) can we make this monochromatic subgraph? I will discuss some partial answers for various types of subgraphs, including paths, locally finite bipartite graphs, and others.

Strong Equitable Choosability of Graphs

Jeffrey Mudrock
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Joint work with: Hemanshu Kaul, Michael Pelsmajer, and Benjamin Reiniger.

The study of equitable coloring began with a conjecture of Erdős in 1964, and it was formally introduced by Meyer in 1973. An equitable k-coloring of a graph G is a proper k-coloring of G such that the sizes of the color classes differ by at most one. In 2003 Kostochka, Pelsmajer, and West introduced a list analogue of equitable coloring, called equitable choosability. Specifically, given lists of available colors of size k at each vertex of a graph G, a proper list coloring is *equitable* if each color appears on at most $\lceil |V(G)|/k \rceil$ vertices. Graph G is said to be *equitably k-choosable* if such a coloring exists whenever all the lists have size k. In this talk we introduce a new list analogue of equitable coloring which we call *strong equitable choosability*. We present some basic facts about this concept, completely characterize strongly equitably 2-choosable graphs, and discuss strong equitable choosability of graphs with small order.

Ramsey numbers of interval 2-chromatic ordered graphs

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Joint work with: Douglas West.

An *ordered graph* G is a graph together with a specified linear ordering on the vertices, and its interval chromatic number is the minimum number of independent sets consisting of consecutive vertices that are needed to partition the vertex set. The t-color Ramsey number $R_t(G)$ of an ordered graph G is the minimum number of vertices of an ordered complete graph such that every edge-coloring by t colors contains a copy of G in some color i, where the copy of G preserves the original ordering on G.

We obtain lower bounds linear in the number of vertices for the ordered Ramsey numbers of certain classes of 2-ichromatic ordered graphs using the methodology of Balko, Cibulka, Král, and Kynčl. We also determine the exact value of the t-color Ramsey number for two families of 2-ichromatic ordered graphs.

A Bipartite Party Problem

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Joint work with: Zhenming Bi, Gary Chartrand, and Ping Zhang.

A well-known party problem with a graph theory connection is the following: What is the smallest number of people who must be present at a party such that there are three mutual acquaintances or three mutual strangers? This problem has many generalizations. Here, we consider the following bipartite party problem along with some of its extensions and its graph theory connection: at a party with six girls, what is the smallest number of boys who must be present at the party to guarantee that there are three girls and three boys such that (1) each of the three girls is an acquaintance of each of the three boys or (2) each of the three girls is a stranger to each of the three boys?

A generalization of Stirling numbers arising from chordal graphs

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Joint work with: David Galvin.

For any graph G on n vertices, we can define the graph Stirling number of the second kind $\left\{ {G \atop k} \right\}$ to be the number of ways of partitioning the vertex set of G into k nonempty independent sets. If G is itself an independent set, this gives rise to the classical Stirling numbers of the second kind, $\left\{ {n \atop k} \right\}$, and the inverse of the matrix $\left(\left\{ {m \atop k} \right\}\right)_{0\le m,k\le n}$ has its (m,k)-entry equal to $(-1)^{m+k}\left[{m \atop k} \right]$, where $\left[{m \atop k} \right]$ are the unsigned Stirling numbers of the first kind. Another known property of the matrix $\left(\left\{ {m \atop k} \right\}\right)_{0\le m,k\le n}$ is that it is totally nonnegative — all minors are nonnegative.

We generalize these classical results. We consider the matrix $\left(\left\{ {G_m \atop k} \right\}\right)_{0\le m,k\le n}$, where G has vertices $v_1, \dots, v_n$, and G_m is the subgraph induced by $v_1, \dots, v_m$. We show that if G is chordal and $v_1, \dots, v_n$ are in a perfect elimination order, then $\left(\left\{ {G_m \atop k} \right\}\right)_{0\le m,k\le n}$ is totally nonnegative, and its inverse displays the same checkered sign pattern as the inverse of $\left(\left\{ {m \atop k} \right\}\right)_{0\le m,k\le n}$.

An Extremal Problem on the Lights Out Game

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We study the following generalization of the game "Lights Out". We begin with a graph G whose vertices are labeled with elements of $\mathbb{Z}_\ell$ for some $\ell \geq 2$. We play the game by toggling the vertices. Each time the vertex v is toggled, we add 1 to the labels of both v and each of its adjacent vertices. The game is won when each vertex has label 0. A graph G is called *always winnable over* $\mathbb{Z}_\ell$ if for every vertex labeling of G, the Lights Out game can be won. In this talk, we consider the problem of determining always winnable graphs of a given order that have maximum size. We present upper and lower bounds on the size of such extremal graphs along with some other results related to the problem.

Rainbow Spanning Trees in Properly Edge-Colored Graphs

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Joint work with: Hung-Lin Fu, Yuan-Hsun Lo, and Chris Rodger.

A spanning tree of a properly edge-colored graph is rainbow provided that each of its edges receives a distinct color. In 1996, Brualdi and Hollingsworth conjectured that if K_{2m} is properly $(2m-1)$- edge-colored, then the edges of K_{2m} can be partitioned into m rainbow spanning trees, except when $m = 2$. In this talk, we will look at an inductive argument which constructs approximately $\sqrt{m}$ rainbow edge-disjoint spanning recursively in any properly edge-colored K_{2m}. We'll also look extending this algorithm to insist on certain structural characteristics within the trees.

On the Zero-Forcing Polynomial of a Graph

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The minimum rank of a simple graph G (denoted $m(G)$) is defined as the smallest possible rank over all symmetric real matrices whose ijth entry (for $i \neq j($ is nonzero precisely when $ij \in E(G)$. The minimum size of a zero-forcing set of G has been shown to form a bound for $m(G)$, and so we study the graph polynomial $\mathcal{Z}(G, x) = \sum z(G, k)x^k$ where $z(G, k)$ is the number of zero-forcing sets of G of order k. We compute the zero-forcing polynomials of several classes of graphs, properties of the polynomials for other classes of graphs, and structural results and properties of the zero-forcing polynomials of general graphs.

New results on the minimum number of distinct eigenvalues of graphs

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Joint work with: Beth Bjorkman, Leslie Hogben, Scarlitte Ponce, and Theodore Tranel.

The minimum number of distinct eigenvalues for a graph G, $q(G)$, is the minimum number of distinct eigenvalues over all real symmetric matrices whose off-diagonal entries correspond to adjacencies in G, denoted $\mathcal{S}(G)$. This relatively new parameter is of interest due to its relationship to the inverse eigenvalue problem which tries to determine all possible spectra for $\mathcal{S}(G)$. New results to be presented include bounds on $q(G)$ for graph products such as Cartesian products and strong products, as well as joins and block-clique graphs.

Guessing numbers of graphs

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The guessing number problem is the following. What is the largest family of colorings of a graph such that the color of each vertex is determined by its neighborhood? This problem is equivalent to finding protocols for network coding. I will discuss results on general graphs, and recent asymptotic results for odd cycles, which is joint work with Ross Atkins and Fiona Skerman.

$(3, 1)$-colorings of 4-regular graphs

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Joint work with: Anton Bernshteyn, Omid Khormali, Ryan R. Martin, Jonathan Rollin, Songling Shan, and Andrew J. Uzzell.

Using edge cuts and Tutte's 1-Factor Theorem, Tashkinov (1982) settled the Berge–Sauer conjecture: Every 4-regular simple graph does indeed contain a 3-regular subgraph. The question remains open, however, for 4-regular pseudographs—that is, for graphs with loops and multi-edges allowed. Bernshteyn (2014) introduced the use of edge-colorings as an approach to this problem, proving that a 4-regular pseudograph contains a 3-regular subgraph if and only if it admits an ordered $(3, 1)$-coloring. A $(3, 1)$-*coloring* of a 4-regular graph is an edge coloring in which every vertex v is incident to 3 edges of a color i_v and 1 edge of a different color j_v. The coloring is *ordered* provided the colors are linearly ordered and $i_v < j_v$ at every vertex v. We completely characterize $(3, 1)$-colorable pseudograph, though a characterization of the ordered-$(3, 1)$-colorable pseudograph remains at large.

Anti-van der Waerden number of 3-term arithmetic progressions

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Joint work with: Zhanar Berikkyzy and Michael Young.

A set is *rainbow* if each element of the set is a different color. A coloring is *unitary* if at least one color is used exactly once. The *anti-van der Waerden number* of the integers from 1 to n, denoted by $\text{aw}([n], 3)$, is the least positive integer r such that every exact r-coloring of $[n]$ contains a rainbow 3-term arithmetic progression. The *unitary anti-van der Waerden number* of the integers from 1 to n, denoted by $\text{aw}_u([n], 3)$, is the least positive integer r such that every exact unitary r-coloring of $[n]$ contains a rainbow 3-term arithmetic progression. Bounds for the anti-van der Waerden number and the unitary anti-van der Waerden number on the integers have been established. The exact value of the unitary anti-van der Waerden number of the integers is equal to the anti-van der Waerden number of the integers and these are given by $\text{aw}([n], 3) = \text{aw}_u([n], 3) = \lceil \log_3 n \rceil + 2$.

The Maximum Decycling Number of a Graph

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The *decycling number* of a graph G (denoted $\nabla(G)$) is the smallest size of a subset S of the vertex set $V(G)$ such that G - S is acyclic. A decycling set of order $\nabla(G)$ is minimal with respect to the decycling property. In this talk we will explore the question: Are there larger subsets of the vertex set which are also decycling sets, yet minimal with respect to that property?

We define a ∇-critical set S of a graph G to be a subset of the vertex set which is a decycling set, but for every vertex v in S, G - (S - v) contains a cycle. The *maximum decycling number* of a graph G (denoted $\nabla_m(G)$) is the maximum order of a ∇-critical set of G.

Switching of Covering Codes

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Joint work with: Patric R. J. Östergård (Department of Communications and Networking, Aalto University School of Electrical Engineering, Finland).

Let Q_n denote the n-dimensional cube. Define the (closed) r-ball centered at vertex y of Q_n to be the set of vertices whose Hamming distance from y is at most r. If a set D of vertices of Q_n has the property that every vertex of Q_n is in the r-ball centered at some vertex of D, we say that D is a (binary) *covering code* of length n and covering radius r. (Equivalently, D is a dominating set of the rth power of Q_n.)

A *switch* of a covering code is a change in the same coordinate of each codeword that gives a covering code of the same radius.

We here study the *semiflip*, which is a switch that replaces the same coordinate of each codeword with a parity check bit. This transformation was previously studied by Struik, who showed that it is a switch, and by Miller and Perkel, who used it to find the graph automorphism group of each power of Q_n.

Finite products of semiflips are *semiautomorphisms* of Q_n. Semiautomorphisms do not preserve Hamming distance, but surprisingly they do send balls to balls, and in fact are characterized by this property.

Main Theorem *For $n \geq 3$, the following conditions on a permutation ψ of $V(Q_n)$ are equivalent:*
(A) ψ is a semiautomorphism;
(B) For every r, $0 \leq r \leq n$, ψ permutes the set of r-balls of Q_n;
(C) For some r, $1 \leq r \leq n-2$, ψ permutes the set of r-balls of Q_n.

Semiautomorphisms give interesting connections between covering codes. We show some relationships among optimal codes of size at most 7, and of codes of length 8 and covering radius 1. Semiautomorphism classes of these codes are found.

Posters

- Abhay Goel, Kalamazoo College. *A generalization of the Bestvina-Brady construction.*
- Ellen Grove, Samantha Law, Morgan Oneka, Mikaela Wyatt, Grand Valley State University. *The MIGHTY-est Chicken: A Graphical Investigation of Chicken Pecking Orders.*
- Casey Koch-LaRue, Grand Valley State University. *Geometries from Groups.*
- Grace McMonagle, Grand Valley State University. *Embeddability of Partial Latin Squares in the Cayley Table of Groups.*
- Maddie Rainey, Grand Valley State University. *Quantifying the Variability of Baseflow of Watersheds for the Chesapeake Bay.*
- Rebecca Robinson, University of Michigan – Flint. *Intersection Graphs of Maximal Convex Subpolygons of k-Lizards.*
- Evan Runburg, Abe Yeck, Ethan Zewde, Michigan State University. *Quiver Mutation and Maximal Green Sequences.*
- David Shane, Grand Valley State University. *Predicting Separability from Partial Preference Matrices.*
- Christopher St. Clair and George Brooks, Saginaw Valley State University. *Induced Colorings of Graphs.*